About an ether interpretation for the Einstein equations of general relativity

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Following Bell's paper "how to teach special relativity", we argue that a generalization of the Lorentz ether interpretation of the Einstein equations of general relativity may have a similar pedagogical value as the Lorentz ether interpretation for special relativity: It will give better intuitions to those who learn it.

The Lorentz ether interpretation used here is the limit of an alternative ether theory of gravity, which gives the Einstein equations of GR in harmonic coordinates, together with an interpretation of the gravitational field in terms of density, velocity and stress tensor of an ether.

Different examples where this "Lorentz ether" gives better, less confusing intuitions than a curved spacetime are considered. It is argued that using the Lorentz ether would decrease the problems with "ether cranks", giving them the possibility to accept GR physics without giving up their prejudices about "true" space and time. The limitations of the Lorentz ether – which requires a fixed background – can be used to obtain a better understanding of the revolutionary character of the spacetime interpretation and background independence of GR.

Keywords: gravity, Lorentz ether

I. PREFACE

Many people think that the Michelson-Morley experiment was the death penalty not only for particular ether theories, but the ether in general. Much less people know that there is the Lorentz ether, an ether interpretation of special relativity (SR), which predicts exactly the same as SR for the Michelson-Morley experiment as well as every other experiment. A few people have heard about Einstein's Leyden lecture [7], where Einstein embraced the notion of the ether even for general relativity (GR). But it is almost unknown that the Lorentz ether interpretation of special relativity can be generalized in a quite simple way into a Lorentz ether interpretation of the Einstein equations of general relativity in harmonic coordinates. In this interpretation, the gravitational field describes the density, the velocity and the stress tensor of some ether, and the harmonic coordinate conditions are interpreted as the continuity and Euler equations for this ether. One aim of this paper is simply to give an introduction to this Lorentz ether interpretation of relativistic gravity.

This includes a consideration of the main difference between the Lorentz ether and the spacetime interpretation – the existence of a background. This background has several consequences: It excludes solutions with nontrivial topopology, as well as with causal loops, and leads to different notions of completeness and of homogeneity. At least in principle, this makes the ether interpretation even a different physical theory: Observing a nontrivial topology or a causal loop would falsify the ether interpretation, but not the spacetime interpretation.

The focus of this paper is, instead, a pedagogical one, following Bell's remark that "we need not accept Lorentz's philosophy to accept a Lorentzian pedagogy" [3]. The main point is that, independent of the arguments in favor or against a particular interpretation, it would be useful to know (and to teach) both interpretations.

Some of the arguments have general character: Knowing different interpretations allows much better to distinguish physics (what all interpretations share) from metaphysics (what can differ). Then, different interpretations have very different problems, and attempts to solve them provide different starting points for further theory development. This point is illustrated by an ether theory of gravity proposed by the author [14] with the aim to improve this ether interpretation.

Other points are specific to the ether interpretation. We will find that the ether interpretation suggests in many questions intuitions quite different from those suggested by the spacetime interpretion, and those different intuitions often appear more adequate. This supports Bell's "impression that those with a more classical education, knowing something of the reasoning of Larmor, Lorentz, and Poincare, as well as that of Einstein, have stronger and sounder instincts" [3].

A specific advantage of the ether interpretation is that it is, contrary to the curved spacetime, not in conflict with common sense and classical Newtonian concepts about space and time. This allows to teach the mathematics and physics of relativistic gravity without a necessity to introduce any new ideas about space and time. Even the key point of the spacetime interpretation – the unobservability of the background – can be teached without questioning its very existence, but, as done in the ether interpretation, by explaining the unobservability by distortions of the rulers and clocks by the ether. This is not in conflict with common sense, which is used to handle distortions of measurement instruments. Then it can be left to decision of the pupils if they prefer to follow the common sense compatible ether interpretation, or accept that, once the background does not have any directly observable consequences, it makes no sense to assume its existence too.

This would allow to solve the problems of "ether cranks", who reject even special relativity as "logically wrong" and explain the success of relativistic physics with anti-scientific conspiracy theories. What they reject is, essentially, the spacetime interpretation. Rejecting the spacetime interpretation means, at least this is how it looks today, to reject relativistic physics completely. If they would know that relativistic physics is compatible with classical ether metaphysics, they probably would not become anti-scientific cranks, but, instead, follow the ether interpretation and reject only the spacetime interpretation.

II. THE LORENTZ ETHER INTERPRETATION OF THE EINSTEIN EQUATIONS

The equations of the Lorentz ether interpretation are the Einstein equations of GR in harmonic coordinates. The harmonic coordinate condition is, different from the spacetime interpretation of GR, not only a quite arbitrary coordinate condition without any physical importance, but an additional physical equation. It distinguishs the preferred coordinates $\mathfrak{x}^0 = \mathfrak{t}, \mathfrak{x}^i$ of a Newtonian background:

$$\square \mathfrak{x}^\nu = \partial_\mu (g^{\mu\nu}\sqrt{-g}) = 0. \tag{1}$$

Given that the harmonic condition has already the form of four conservation laws for a symmetric tensor, it seems natural to compare it with the conservation laws we know from condensed matter theory – the continuity and Euler equations:

$$\partial_t \rho + \partial_i \rho v^i = 0 \tag{2}$$
$$\partial_t \rho v^j + \partial_i (\rho v^i v^j + \sigma^{ij}) = 0 \tag{3}$$

for density ρ, velocity v^i and stress tensor σ^{ij} of some condensed matter. This suggests the following identifications:

$$g^{00}\sqrt{-g} = \rho, \quad g^{0i}/g^{00} = v^i, \quad g^{ij}\sqrt{-g} = \rho v^i v^j + \sigma^{ij}. \tag{4}$$

To obtain a positive density $\rho > 0$, all we have to require is that $\mathfrak{x}^0 = \mathfrak{t}$ is time-like. If the other coordinates $\mathfrak{x}^i$ are space-like, the stress tensor σ^{ij} would be negative-definite.

As in the special-relativist Lorentz ether, the preferred coordinates define a classical Newtonian world with absolute space and time. The spatial coordinates $\mathfrak{x}^i$ are standard Cartesian coordinates of absolute space, and the time coordinate $\mathfrak{t}$ defines absolute time. Similarly we are unable to measure absolute distances and times with our rulers and clocks, because they are distorted by the ether: What they measure are the distances defined by the spacetime metric $g_{\mu\nu}(\mathfrak{x}, \mathfrak{t})$. An important difference is that the ether is no longer motionless, homogeneous, and incompressible, but follows the usual equations for compressible condensed matter.

III. THE PEDAGOGIC VALUE OF THE MATHEMATICS OF THE ETHER INTERPRETATION

The mathematics required in the ether interpretation are important, useful to be teached anyway. The harmonic coordinates, proposed by de Donder [5] and Lanczos [10], simplify the Einstein equations in a very essential way: They obtain the form

$$G^{mn} = g^{ab}\partial_a\partial_b g^{mn} + F^{mn}(g^{pq}, \partial_k g^{pq}) \tag{5}$$

without any mixing of different components of g^{mn} in the second order derivative terms. They are probably the most often used coordinate condition in GR. In particular, they play a key role in "unreasonable effective" post-Newtonian approximation [19]. Based on this important simplification, Fock [8] has argued in favor of harmonic coordinates being natural preferred coordinates. Indeed, what makes classical Cartesian coordinates preferred in classical physics is that the equations of physics obtain an especially simple form. The same can be said for harmonic coordinates too.

One important argument in this direction is that the first local existence and uniqueness theorems for the Einstein equations have been proven by Bruhat in harmonic coordinates [4]. The point mentioned above that the highest order terms no longer mix in harmonic coordinates played the decisive role here.

The other part of the mathematical apparatus used in the Lorentz ether interpretation is the decomposition of the gravitational field $g^{mn}\sqrt{-g}$ into a scalar field (ρ), a three-vector field (v^i), and a three-metric (σ^{ij}), which is the result of the choice of one time-like harmonic coordinate as the time. This is a variant of the ADM decomposition, which was necessary to define a Hamiltonian formulation for GR [1].

So, even if teaching the Lorentz ether interpretation would require some additional mathematics, all these additional elements – the harmonic coordinate condition as well as the ADM decomposition – are important contributions to the mathematics of GR, are proposed and widely used without any connection to the Lorentz ether, and have their own value. This allows to make two points: On the one hand, teaching the Lorentz ether gives a nice motivation to teach these additional mathematics. On the other hand, all the mathematics used in the Lorentz ether have been developed and widely used independently, which suggests that the interpretation is in itself quite natural.

IV. THE LIMITATIONS OF THE LORENTZ ETHER

It is not without reason that I have named the Lorentz ether an "interpretation of the Einstein equations" instead of an "interpretation of GR". The reason is that the Lorentz ether interpretation has some limits.

Locally, it is always possible to define the ether interpretation. In particular, one can always introduce locally some harmonic coordinates. This is a side effect of the point that the proof of local existence and uniqueness for the Einstein equations given by Bruhat [4] is given in harmonic coordinates. But the harmonic equation, given in terms of the wave equation $\Box\mathfrak{x}^\nu = 0$, is also simple enough to see this. Then, to choose among such local harmonic coordinates one time-like and three space-like coordinates is also unproblematic, given that linear combinations of harmonic coordinates are harmonic too.

The situation becomes different if one looks at global solutions. The global solution from point of view of the Lorentz ether interpretation requires a very special additional global object – a Newtonian background of absolute space and time. This additional requirement makes the Lorentz ether, even if it uses the same Einstein equations as GR, and the harmonic coordinate condition widely used in GR as the only additional equation, a different physical theory. This is a nice and pedagogically useful illustration of the general thesis that the equations are not all what defines a physical theory.

Each of the following points allows to demonstrate important physical differences between the Lorentz ether and the curved spacetime of GR. Only by understanding the boundaries of the Lorentz ether allows to understand the fundamentally new, revolutionary elements of the spacetime interpretation of GR, understand the meaning of what is usually hidden behind the phrase "background freedom" and "equivalence principle".

So, let's list the important physical limitations of the Lorentz ether in comparison with GR:

A. Additional degrees of freedom

One can have different harmonic coordinates for the same metric. If one looks at them as solutions of the Lorentz ether, these different coordinates define different solutions, with different ether densities, velocities and stress tensors. It is easy to classify these additional degrees of freedom – these are simply four scalar fields, the preferred coordinates, which are described by the harmonic equation.[1] From point of view of GR, we have the same solution, simply in different coordinates – but different coordinates are, from GR point of view, physically irrelevant, because unobservable.

B. Trivial topology

The Lorentz ether depends on a given choice of harmonic coordinates, and defines the ether variables only in the domain of definition of these coordinates. Thus, a flat background is presupposed. So, it cannot be used to describe solutions with nontrivial topology – except for the trivial case of solutions on a torus, which can be considered as periodic solutions on $\mathbb{R}^4$. Thus, the Lorentz ether will not be helpful at all for the consideration of wormholes and other solutions where a nontrivial topology is important.

C. Different notions of completeness

A solution is "complete" from point of view of the Lorentz ether if it is defined for all all values $-\infty < \mathfrak{x}^\mu < \infty$. This notion of completeness is different from the notion of completeness of GR, which requires that the metric is complete. This difference becomes physically relevant for the gravitational collapse of a star. To define the harmonic coordinates, we have to fix their initial values before the collapse starts. Here we can use the uniqueness theorem proven in sec. 93 of [8]. For these initial values, the time coordinate becomes infinite where the star reaches the horizon [18].[2] For the Lorentz ether, it would be clear that this solution is complete – the true, fundamental time would be harmonic time $\mathfrak{t}$. Of course, even in the Lorentz ether an infalling clock would reach infinity in finite "proper time", but this would be, for the Lorentz ether, only an extremal case of clock time distortion, where the clock is de-facto completely frozen, its speed goes exponentially down.

The GR picture is, of course, completely different. The harmonic time becoming infinite means nothing at all, a complete solution has to be complete from the point of the metric, the time which physically matters is proper time, and the part behind the horizon is clearly part of the complete GR solution.

D. Global hyperbolicity

Even more radical is what happens with Gödel's rotating universe solution. This solution contains causal loops, thus, as a consequence does not contain any global

[1] Of course, there is a difference between such scalar fields and coordinates – not every configuration of four scalar fields defines a valid system of coordinates.

[2] A simple ansatz for a harmonic metric inside the star allows to illustrate this:

$$ds^2 = (1 - \frac{M\cdot\partial M/\partial\mathfrak{r}}{\mathfrak{r}})(\frac{\mathfrak{r}-M}{\mathfrak{r}+M}dT^2 - \frac{\mathfrak{r}+M}{\mathfrak{r}-M}d\mathfrak{r}^2) - (\mathfrak{r}+M)^2 d\Omega^2. \tag{6}$$

where $\mathfrak{r} = \sqrt{\sum(\mathfrak{r}^i)^2}$ is the radius defined by harmonic spatial coordinates and $M(\mathfrak{r}) < \mathfrak{r}$ a function not decreasing with $\mathfrak{r}$. The complete solution for a collapsing star could, in principle, look similar to (6), with some $M(\mathfrak{r},\mathfrak{t}) < \mathfrak{r}$ and reaching $\mathfrak{r}$ only in the limit $\mathfrak{t}\to\infty$. For $M(\mathfrak{r},\mathfrak{t})$ only the spatial coordinates are harmonic, but one could imagine that $\mathfrak{t}$ remains, for an adiabatic collapse, a reasonable approximation. Note also that this ansatz does not describe the general case, which inside the matter would have to depend also on a second function, the pressure.

time-like coordinate. Thus, it does not allow a global Lorentz ether picture. Every choice of a harmonic time coordinate would contain regions where $\mathfrak{t}$ is not time-like, which, in terms of the Lorentz ether, would mean a negative ether density $\rho < 0$. The parts which negative density would have to be rejected as unphysical in a Lorentz ether theory. A natural modification would be an ether which tears, leaving some regions with $\rho = 0$. Such a theory would require new physics, namely boundary conditions for the ether where the ether density becomes zero, and completely unknown physics (maybe some complete vacuum) in the regions where the ether density is below zero.

E. Different meaning of homogenity

A last remarkable effect arises if one considers the FLRW solutions: The solutions with non-zero curvature define an ether which is clearly inhomogeneous. As a consequence, only the flat FLRW solutions define a homogeneous Lorentz ether. Of course, one can define harmonic coordinates also for the curved FLRW solutions – but these solutions cannot define a homogeneous universe. Instead, from point of view of standard GR, the curved FLRW solution define homogeneous solutions.

This modifies the theory only implicitly: If we add the cosmological hypothesis that the universe is, on the large scale, approximately homogenous, we would have to reject in the Lorentz ether the FLRW models with non-zero curvature, but not in GR.

F. The consequences for empirical content of both interpretations

Some of these differences between the two interpretations could, at least in principle, be used to find out that the ether Lorentz ether interpretation is wrong. An interesting point is that this works only only in one direction: Every solution of the ether would be also a solution of GR. It would be, at worst, incomplete as a GR solution – but this would be nothing we would be able to find out if we would live in the part of the solution which is described by the ether too. From the point of view of scientific methodology, that means that there are more possibilities to falsify the ether interpretation than the spacetime interpretation. This means, the empirical content of the ether interpretation is higher, and it would have to be preferred by Popper's criterion, as long as it is not yet actually falsified.

Up to now, it is not falsified.

V. HOW TO USE THE LIMITATIONS OF THE LORENTZ ETHER TO TEACH GR

This impressive list of limitations of the Lorentz ether illustrates that, even if the Lorentz ether uses the same equations as GR, it defines a very different physical theory. A theory with four additional scalar fields, without wormholes, causal loops, and the part behind the horizon, without homogeneous curved universes is, at least in principle, a different physical theory.

It is also important to understand what all the differences have in common: It is the existence of an additional structure – the Euclidean spacetime $\mathbb{R}^3 \times \mathbb{R}$ as a global fixed background – which makes the difference. It gives the four additional degrees of freedom, which correspond to the four additional equations, it introduces its symmetry into the meaning of "homogeneous universe", it is incompatible with nontrivial topologies, with the causal loops of the Gödel universe, and with some parts of complete GR solutions.

Instead, GR does not contain such a background, it is a *background-independent* theory. It is background-independence, which is considered by many quantum gravity researchers, for example Rovelli [12] and Smolin [16], as the most important, decisive property of general relativity. It has deep philosophical roots going back to the dispute between Newton, who defended the concept of absolute space, and the relationalist point of view of Leibniz, who rejected absolute space: GR is a completely relational theory (see [12] [16] about this).

The Lorentz ether is useful to illustrate this difference, because the Lorentz ether can be understood as the theory which realizes the Newtonian concept of absolute space and time in the domain of relativistic gravity. This, in particular, makes clear that these conceptual differences do not depend on the Einstein equations, which are used in both theories.

The usefulness of the Lorentz ether can be illustrated by considering another possible error – one made by Einstein himself, who initially thought that the covariance of the Einstein equations is the distinguishing property of GR. But, as criticized by Kretschmann [9], all physical theories can be written in a covariant form. This point can be nicely illustrated by the Lorentz ether: It is a theory with preferred coordinates – the harmonic coordinates. But the harmonic condition can be easily given a covariant form: The harmonic equation, which defines the preferred coordinates, can be written as a covariant equation $\Box\mathfrak{t}(x) = \Box\mathfrak{x}^i(x) = 0$ for the $\mathfrak{t}(x), \mathfrak{x}^i(x)$ expressed as functions of general coordinates x.

Einstein has accepted this argument, but argued that, nonetheless, the covariance principle has heuristic power, if combined with the preference of a simpler theory: A covariant formulation of Newtonian theory would be, he suggests, so complex that it would have to be rejected if not for theoretical, then for practical reasons[3], an argument which is supported, in particular, also in §12.5 of MTW [11]. The Lorentz ether also allows to illustrate this: The covariant formulation contains, indeed, the four additional fields – the preferred coordinates $\mathfrak{t}(x), \mathfrak{x}^i(x)$ – with additional equations for them.[4]

Which particular example is the most useful one to illustrate particular points remains an interesting question. I would suggest that the FLRW solution with positive curvature is an ideal example for a solution with nontivial topology: $S^3 \times \mathbb{R}$ is a sufficiently simple topology, and, moreover, a viable candidate for our universe. To illustrate global hyperbolicity and the impossibility for GR to enforce it, the Gödel universe seems ideal: It is a solution defined on $\mathbb{R}^4$, thus, no non-trivial topology confuses the picture. Moreover, Newtonian intuitions predict that at some distance the centrifugal forces would become too large so that the rotating ether would tear. To illustrate the different notions of completeness of a solution, one can use the gravitational collapse, where the Lorentz ether excludes the part after horizon formation, so that the complete GR solution is greater than the complete Lorentz ether solution. The reverse situation can be illustrated with the Big Bang singularity of GR. Here, the GR solution is not complete, but ends in a singularity in finite time. The harmonic coordinates for a flat FLRW universe are $ds^2 = a^6(\mathfrak{t})d\mathfrak{t}^2 - a^2(\mathfrak{t})(\sum(d\mathfrak{x}^i)^2)$, so that $d\mathfrak{t} = a^{-3}(\tau)d\tau$. Thus, if the Big Bang singularity is in proper time of the form $a(\tau) \sim \tau^\alpha$ with $\alpha \geq \frac{1}{3}$, then there will be no Big Bang singularity at all in terms of harmonic time $\mathfrak{t}$, because the solution extends to $\mathfrak{t} \to -\infty$.

VI. BEYOND THE ETHER INTERPRETATION

The aim of this paper is not the discussion of the Lorentz ether as an alternative theory for modern physics – in this case, one would consider the violation of Bell's inequality (as an argument that a hidden preferred frame is necessary anyway if one wants to preserve causality), or on the problem of quantization (which, once we know how to quantize condensed matter theories on a Newtonian background, would be at least conceptually much less problematic) or its connection to quantum field theory (where a background is helpful), or consider the question if there is more behind the various mathematical similarities between condensed matter theories and fundamental physics used in the Wilsonian approach to renormalization [20] or in the large body of research in analogue gravity [2], which has been followed Unruh's idea that in acoustic analogons of black holes where will be an effect similar to Hawking radiation [17].

But one development of the Lorentz ether interpretation is worth to be mentioned here, because it teaches an interesting point why having different interpretations may be important in fundamental physics: Interpretations have weak points, and improving the interpretation by solving such a problem can require a modification of the theory itself, leading to a different theory, with different equations.

The Lorentz ether interpretation has an interesting problem which does not exist in the spacetime interpretation: It does not have an Euler-Lagrange formalism. There is an Euler-Lagrange formalism for GR, with the Einstein-Hilbert Lagrangian, but this Lagrangian gives only the Einstein equations, it does not give the harmonic coordinate condition. In the spacetime interpretation this is unproblematic, because the coordinates are arbitrary, and a coordinate condition is not a physical equation. But in the Lorentz ether interpretation, the harmonic condition is a physical equation. Thus, it would have to be an Euler-Lagrange equation, if there exist a Lagrange formalism for the theory.

Fortunately there is a simple way to solve this problem. There exist a well-known Lagrangian which gives the harmonic wave equation, namely the standard Lagrangian for a scalar field, and the harmonic coordinate condition is nothing but a harmonic wave equation for the preferred coordinates $\Box\mathfrak{x}^\nu = 0$. So, all we have to do to obtain the harmonic condition as an Euler-Lagrange equation is to add the corresponding terms to the GR Lagrangian:

$$L_{GR} \to L_{GR} + \Xi_\alpha g^{\mu\nu}\sqrt{-g}\partial_\mu\mathfrak{x}^\alpha\partial_\nu\mathfrak{x}^\alpha. \tag{7}$$

But this changes also all the other equations: There appear now some new terms in the Einstein equations which introduce some dependence of the equations on the preferred coordinates $\mathfrak{x}^\nu$. The dependence is a quite harmless one, because it does not contain derivatives of the metric and has, therefore, cosmological character. Moreover, it depends on the size of the parameters Ξ_α. All we need is that they are not exactly zero, but they can be arbitrary small. But, nonetheless, the equations of the theory are different now, so that we have obtained already a new, non-covariant theory of gravity. This theory of gravity has been introduced in [14], and as the author, I can tell

[3] "Wenn es nämlich auch richtig ist, daß man jedes empirische Gesetz in allgemein kovariante Form muß bringen können, so besitzt das Prinzip a) doch eine bedeutende heuristische Kraft ... Von zwei mit der Erfahrung vereinbarten theoretischen Systemen wird dasjenige zu bevorzugen sein, welches vom Standpunkte des absoluten Differentialkalküls das einfachere und durchsichtigere ist. Man bringe einmal die Newtonsche Gravitationmechanik in die Form von absolut kovarianten Gleichungen (vierdimensional) und man wird sicherlich überzeugt sein, daß das Prinzip a) diese Theorie zwar nicht theoretisch, aber praktisch ausschließt!" [6] p. 242

[4] It also illustrates that the difference is not as impressive as it seemed to Einstein, and looks insufficient to exclude theories with preferred coordinates. Moreover, the value of this argument remains questionable: If two theories look comparably ugly in general coordinates, but one looks much easier in particular preferred coordinates, this is a reasonable simplicity argument. Einstein's argument requires to ignore it, without giving a justification. See also Fock's argumentation [8] about preferred coordinates.

that this was indeed the motivation to introduce these additional terms.

In the new theory, some problems created by the covariance of GR disappear. We obtain a straightforward non-degenerated Hamilton formalism. The Lagrange formalism gives the "action equals reaction" principle, so that the additional degrees of freedom become observable in principle (even if only like dark matter). The Noether theorem gives in the classical, straightforward way local energy and momentum conservation laws, even in two variants: One variant are the harmonic conditions. Given that the tensor $g^{\mu\nu}\sqrt{-g}$ appears also in the equations for the gravitational field, one can use these equations to give this tensor a more conventional form:

$$\begin{aligned} T^\mu_\nu\sqrt{-g} &= (T_m)^\mu_\nu\sqrt{-g} \\ &\quad + (8\pi G)^{-1}\left((\Lambda + \Xi_\alpha g^{\alpha\alpha})\delta^\mu_\nu - G^\mu_\nu\right)\sqrt{-g} \end{aligned} \tag{8}$$

The limit $\Xi_\alpha \to 0$ is interesting here: The harmonic equation ceases to be an Euler-Lagrange equation, and can survive only as a coordinate condition. The second form simply reduces to an expression which is identically zero because of the Einstein equations. Another unexpected advantage was a derivation of the Lagrangian itself, starting from the assumption that one wants the continuity and Euler equations as Noether conservation laws from some Lagrange formalism. The Lagrangian $L_0 = \Xi_\alpha g^{\mu\nu}\sqrt{-g}\partial_\mu\mathfrak{x}^\alpha\partial_\nu\mathfrak{x}^\alpha$ would be one solution, it gives the conservation laws as Euler-Langrange equations for the preferred coordinates $\frac{\delta L_0}{\delta\mathfrak{x}^\nu} = \Xi_\nu\Box\mathfrak{x}^\nu$. For any other Lagrangian, the difference should not give any new terms which would distort the equations. So, $\frac{\delta L-L_0}{\delta\mathfrak{x}^\nu} = 0$, thus, all the other terms have to be covariant, which is what distinguishes the GR Lagrangian.

But these differences are not the point we want to make here. The point is that different interpretations have different problems. The problem of the ether interpretation – that the equations where not Euler-Lagrange equations – did not even exist in the spacetime interpretation, so there would be no reason to solve it in the spacetime interpretation. Thus, the theory of gravity which solves this problem would not have been developed. If the theory is worth anything, the future will show – the point is that this is already a physical question, because the theory is a physically different theory, with different equations.

VII. COMPARISON OF INTUITIONS RELATED WITH THE TWO INTERPRETATIONS

Let's now compare the intuitions related with the two different interpretations.

A. Four-dimensional spacetime vs. distorted rulers and clocks

The spacetime interpretation of GR requires a modification of our world-view on the most fundamental level, namely we have to give up the classical picture of a space, filled with various matter, which changes in time. Instead, we have a strange four-dimensional spacetime, which contains the whole history, of the past as well as the future, and this spacetime does not even contain a well-defined "now". In principle, with time as the forth dimension, this is not that problematic – even Newtonian theory could be presented in a "Newtonian spacetime", which would be simply the set of all events, each defined by its position in space and its time. But the spacetime interpretation rejects the existence of the classical qualitative difference between space and time. It claims that there is no split of this spacetime into space and time. The rejection of absolute contemporaneity is already a serious challenge of classical intuitions.

Here, in comparison, the ether interpretation presents a completely classical picture of reality, which is in no way different from the Newtonian world with absolute space and time. The only non-trivial, new element in comparison with the classical worldview is that rulers and clocks do not measure absolute distances and absolute time, but are distorted by the ether. Fortunately, other technical devices used by humans also have systematic errors, and some of these systematic errors are quite similar, in particular the influence of temperature on rulers made of rigid materials.

There are also differences between the influence of temperature on material rulers and the influence of gravity on clocks and rulers in the ether interpretation: The influence of temperature on a ruler depends on its material, while the influence of the gravitational field is universal, the same for all clocks and rulers. This difference can be used to explain why, as the result, some key properties become unobservable. Here, one can remember how a classical thermometer works: A glass containing some liquid, like mercury, which shows temperature because the volume increase of the liquid is larger than that of the glass. If the volume increase would be the same, the thermometer would not work, and the temperature would become unobservable with thermometers. And, naturally, with the temperature the distortion of the ruler caused by temperature becomes unobservable too, and, as a consequence, the undistorted reference length – classically, the length of the ruler at some reference temperature, becomes unobservable too.

B. Curved spacetime vs. inner stress

Even more strange are the intuitions related with the notion of a curved spacetime. Here, the proposed examples, to be used to guide our intuition, are two-dimensional curved surfaces in three-dimensional space, something far away from a four-dimensional curved spacetime which is supposed to describe our reality. Let's note here that this picture is misleading – a larger-dimensional space which contains the curved surface does not exist in GR itself. But a "curved" manifold without a higher-dimensional space to contain this curve is something hard to imagine, and the "curved spacetime" language does not provide here any help.

The picture of distorted rulers give also a correct intuition that the rules of Euclidean geometry will be violated because of the distortion. This can be illustrated by imagining distance measurements of a circle with rigid rulers, influenced by temperature, if the circle is hot inside. During the measurement of the radius, the ruler will be greater, and the measured distance, therefore, smaller than during the measurement of the circumference. So, one has to expect that $u > 2\pi r$. It is also quite obvious that the non-Euclidean geometry will be a consequence of the inhomogeneity of the distortion. A completely homogeneous distortion would be unobservable, because it would lead to yet another Euclidean geometry.

But it is not only the inhomogeneous distortion of clocks and rulers which the ether interpretation provides. The spatial part of the metric tensor defines, in the ether interpretation, the stress tensor of the ether itself. Now, stress can be created in materials in different ways: On the one hand, simply by a deformation. In this case, the forces try to revert the deformation, to recover the stress-free state. But there is also another possibility: That there is no such stress-free reference state, that every deformation only changes the distribution of the stress, but does not allow to reach a stress-free state. This situation is named "inner stress". Now, it appears that the formula which defines the curvature of the spatial part is exactly the formula which defines the inner stress.

The full four-dimensional stress tensor of the whole spacetime contains also some more components. These other components describe how the stress tensor of the ether changes in time. Despite this, the inner stress gives already a three-dimensional picture, thus, one dimension more than the two-dimensional "curved surface" image, and does not require or suggest any curved embedding into a higher-dimensional entity, thus, is already much closer to the mathematics of the equations.

C. Metric vs. coefficients of the sound wave equation

There is another, more subtle difference between the two intuitions: In the Lorentz ether, one would not consider the spacetime metric as the important fundamental object, given that, according to the interpretation, it is a distorted object. Technically, the ether would be described by density, velocity and stress tensor, and these would be the natural candidates for the basic functions which describe the gravitational field. But that means that the gravitational field would not be described by the metric $g_{\mu\nu}$, but by the tensor $g^{\mu\nu}\sqrt{-g}$. This is the tensor which appears in the harmonic wave equation $\square = \partial_\mu g^{\mu\nu}\sqrt{-g}\partial_\nu$. Or, in other words, the gravitational field is described by the coefficients which appear in the natural wave equation for the ether, the equation for sound waves of the ether.

But this changes the intuition about the nature of Lorentz symmetry. Namely, the Lorentz transformation is a transformation which transforms one solution of the wave equation into another solution of the same wave equation. All one needs to get a completely Lorentz-invariant theory is that all equations are variants of the same basic wave equation - that for sound waves of the ether. In this case, it is clear that one can apply the Lorentz transformation to obtain, for any given solution, another solution. This other solution has a completely different description in terms of true space and time, thus, intuition tells us that it is a physically different solution. On the other hand, there would be no method for the experimenters, which exist in one solution, to distinguish it from the other solutions. Whatever they do, they would observe the same, because the transformation of the solution would transform as their behavior, as their observations, into another solution where they would think that they do the same things and observe the same results.

Similarly, all what holds clocks and rulers together would be ruled by the same wave equation, and, given that the Lorentz transformation would transform a clock or ruler into another clock and ruler, but one moving with a different velocity. Clock time dilation and contraction of rulers are the consequence. So, they appear simply because they are described by solutions of the wave equation, and not at all because reality itself is fundamentally Lorentz-covariant.

So the intuition about Lorentz symmetry is completely different. It is an incomplete symmetry. It does not cover reality, which does not have any Lorentz symmetry, but only observable effects. And even these observable effects will not be covered completely in our intuitions: The intuition paints some picture the ether, and this picture not painted by the ether's sound waves. So, we have here a situation where we have, from the start, to distinguish between reality and what we can observe in this reality, given that we can access only ether sound waves. Moreover, the intuition about the ether suggests also some atomic ether. But in this case near the atomic scale the sound equation will become invalid for the description of the ether, and the symmetry of the wave equation will disappear into nothing.

D. Expanding space vs. shrinking rulers

For the Lorentz ether, the "expanding universe" does not expand at all. The only FLRW metric which is homogeneous in the ether interpretation too is the flat one, and in this case the comoving spatial coordinates are already harmonic. With harmonic time, the FLRW ansatz becomes $ds^2 = a^6(\mathfrak{t})d\mathfrak{t}^2 - a^2(\mathfrak{t})(\sum(d\mathfrak{x}^i)^2)$. Therefore in the ether interpretation of the "expanding universe" everything remains on its place, the ether density remains constant, and the expansion is replaced by an universal shrinking of all rulers.

The same picture is, of course, already suggested by the FLRW coordinates themselves – nobody uses coordinates there the universe expands. But the idea to describe this situation in terms of shrinking rulers instead of an expanding space would, from point of view of GR, be rejected: The rulers are what defines what space is, in the same sense that clocks defines what time is. The choice of the FLRW coordinates is, from GR point of view, irrelevant.

The intuitions related with the "shrinking rulers" picture differ essentially form those of an expanding universe. First, a shrinking rulers picture does not suggest any inhomogeneity, any center. Instead, the expanding universe picture suggests that there is some center, with everything flying away from this center. Nothing in the mathematics as well as in the observable effects suggests any such inhomogeneity, thus, the shrinking rulers picture gives a more adequate intuition regarding the existence of such a center.

The other difference in the intuitions is about the nature of the Big Bang singularity. The expansion picture suggests a point singularity, while the shrinking rulers picture suggests a singularity everywhere, over the whole space. This gives a different intuition about the causal horizon – the region which can be causally influenced from a given point. The expansion picture, where everything starts in the same singular point, suggests no such horizon exists – two comoving trajectories, however far away from each other, start from the same point singularity, which makes it natural to assume that they have been causally connected. The shrinking rulers picture does not make any such suggestion: In the intuitive picture, the comoving trajectories remain at the same "true" distance. The mathematics show that without inflation there will be a quite small causal horizon, thus, will be closer to the intuition of the "shrinking rulers" picture.

E. Relativity vs. preferred frame

A point made by Bell in [3] is that the Lorentzian approach suggests to use only a single frame – the preferred one: "Its special merit is to drive home the lesson that the laws of physics in any one reference frame account for all physical phenomena, including the observations of moving observers. And it is often simpler to work in a single frame, rather than to hurry after each moving object in turn." This lesson will remain valid in the ether interpretation of gravity too. Once there is a preferred system of coordinates, one would intuitively prefer to use it. And develop the intuitions based on it. And, given that one system of coordinates remains sufficient to account for all physical phenomena, this remains to be a good choice.

The spacetime interpretation emphasizes that coordinates do not matter, do not have to matter. But, given that the Lorentz symmetry, resp. general covariance, is quite unintuitive, it is far from obvious that everything is really equivalent if one looks at it in a Lorentz-transformed frame, or from completely different coordinates. So, there is the tendency to look at everything from different points of view, using different frames, related with different observers.

What mathematics tells us that this is only a loss of time – one system of coordinates is sufficient to do all the physics covered by it, and there is no need at all to use different coordinates to describe the experiences of different observers. In fact, we are used to think about what happens around us in terms of the same frame even if we move with different velocities or look at the events from different perspectives. This intuitive "preferred frame" is at rest with the surface of the place we live, but we use the same intuition thinking about the Earth rotating and moving around the Sun. So, the suggestion that different observers would require different frames or different systems of coordinates is in fact misleading our intuition, which is used to take into account observer-dependent, perspective effects.

The ether intuitions corresponds much more to thinking in one system of coordinates, the preferred one. It suggests to think about clock time dilation and contraction of rulers as being real distortions. One can do the same mathematics of Lorentz transformations. But they represent something completely different, namely the errors of a moving observer who, for whatever reasons, thinks he is at rest. All the various errors he makes, interpreting wrongly which clocks go slower, which rulers are contracted, and what is contemporary, conspire together with the error about the own velocity to make the error undetectable. This interpretation raises the problem of explaining this conspiracy, but, once this is understood as a consequence of the universality of the wave equation, there is no longer any motivation to think about representations in non-preferred coordinates.

Thinking about other inertial frames being based on error is helpful for not being mislead by the "twin paradox". In the relativistic interpretation, all three frames involved are on equal foot: The frame of the twin at Earth, the frame of the twin travelling forth, and the one of the twin travelling back. The travelling twin switching, at the point of return, from one two another one seems unproblematic, once both are equally valid.

From point of view of the ether interpretation, the Einstein synchronization makes sense only if you assume you are at rest. If one moves, Einstein synchronization will obviously give wrong results, once the light ray forth and back has to travel different distances, thus, will need different time. Now, once you make this error once, and consistently, you will be unable to detect it. But then it makes no sense at all to switch to another system: Even if the travelling twin doesn't know what is at rest, he knows very well that he cannot at rest all the time, travelling forth as well as back.

VIII. WHAT TO DO WITH ETHER CRANKS?

If one participates in discussions about physics in the internet, one will observe a large number of participants who defend quite problematic ideas about relativistic physics usually named "ether cranks" or "crackpots". Their numbers seem comparable with those of perpetuum mobile constructors and Fermat theorem provers of the past – large enough to cause problems in physics forums in the internet, so that many physics forums see no better defense than to completely forbid discussions about ether theories. The ether cranks,typically laymen with some technical background, usually claim that there are "logical errors" made by Einstein in special relativity, and defend simply classical ether theories. To explain the obvious sociological success of special and general relativity, they have not much choice but to invent anti-scientific conspiracies by evil "relativists", a consequence which makes this group problematic beyond relativistic physics. This group is quite specific – most other physics problems do not attrack such groups of "cranks". This asks for explanation. Why is such a foolish idea, which requires quite absurd conclusions about a conspiracy by "relativists", so attractive for so many people?

The answer seems quite obvious: Relativity as presented in the curved spacetime interpretation is in clear conflict with common sense concepts of space and time, concepts which seem obvious enough. And the evidence presented against the classical concepts of space and time, in popular introductions as well as in textbooks, is not sufficient to convince them that these classical concepts have to be rejected.

Here the ether interpretation of the Einstein equations becomes relevant. It shows that, indeed, the classical concept of space and time is compatible with relativistic gravity. An attempt to present the "ether cranks" empirical evidence that classical ideas of space and time are wrong are, therefore, doomed, and not because the "ether cranks" are that stupid, but because there is no sufficient evidence for this.

Fortunately, the ether interpretation suggests also a solution of this conflict – once the "ether cranks" learn that the classical concepts of space and time are not at all in conflict with relativistic gravity, because there is an ether interpretation of relativistic gravity which supports all these classical concepts, the conflict between the world-view of the ether crank and relativistic gravity simply disappears into a conflict between two interpretations of relativistic gravity.

The conflict between proponents of different interpretations of relativistic gravity is completely harmless – it does not motivate a proponent of the ether interpretation to question the experimental results of a century of modern physics and to invent anti-scientific conspiracy theories, because if one follows the ether interpretation of relativistic gravity there is no disagreement at all about the experimental outcomes. So, the "ether crank" has now a chance to accept all the physics of relativistic gravity without having to give up his preferred metaphysics, and without having to accept curved spacetime metaphysics which he finds unacceptable.

IX. HOW TO TEACH RELATIVITY

Let's consider shortly the question what would be the optimal way to teach relativity.

The actual way to teach relativity is, of course, justified by the historical accident that it was not known, up to now, that there exists a simple, beautiful extension of the Lorentz ether to relativistic gravity. Without a viable ether interpretation for relativistic gravity, the Lorentz ether interpretation for special relativity was, clearly, a dead end, and not worth to be teached.

But this is now known to be wrong. A viable ether interpretation for relativistic gravity exists, and it is quite simple and even beautiful. So, the argument that the ether approach is not viable is invalid.

What would be inappropriate, given that one of the greatest advantages of the ether interpretation is that the pupils do not have to accept the counterintuitive spacetime interpretation, is to start as usual and the tell about the ether interpretation only at the end, as some additional curiosity.

On the other hand, given that the spacetime interpretation is the only one which is actually known, it has to be teached too, and also from the start.

But there is no conflict. In fact, it makes sense to teach both interpretations at the same time. This illustrates their differences, and teaches also, from the start, which parts are physical facts, and which parts are metaphysics, interpretation-dependent.

The same concept of presenting different interpretations can be used already if one intruduces the mathematical apparatus of a curved spatial metric $g_{ij}(x)$, using as examples not only a curved surface, but also the metric as measured by rulers distorted by an inhomogeneous temperature distribution. Similarly, if one introduces the Lorentz symmetry as a symmetry group of the wave equation, one can introduce its application to obtain new solutions for the sound wave equation using the sound wave analogon of the Lorentz symmetry.

The simple and meaningful sequence of teaching would be the following:

- One starts with the Lorentz transformation as a method to construct new solutions out of a known solution of the scalar wave equation. This works for every wave equation, that means, even for sound waves. At least for sound waves it is clear that the newly constructed solution is a qualitatively different solution, and that the transformation plays no fundamental role at all.

- One introduces other examples of wave equations which allow the same method to construct other solutions. In particular, the Maxwell equations, the Dirac equation, and the possibility to add mass terms and pointwise interactions between different fields, with non-abelian gauge fields and gauge fields acting on Dirac fermions as examples – thus, all the fields one needs in the Standard Model of particle physics.

- One considers the hypothetical scenario that all accessible fields follow one of these field equations. In this case, it becomes impossible for the inhabitants of this hypothetical world to find out their absolute velocity.

- One works out some details, in particular the consequences for the behavior of rulers and clocks constructed out of field configurations of such fields, to derive clock time dilation and length contraction.

  Up to now, we are yet in the domain of the Lorentz interpretation - nothing counterintuitive, in contradiction with classical ideas about space and time, has happened yet. But all what is necessary has been already derived. In particular, we already know that the inhabitants of this hypothetical world are unable to identify correctly if they are at rest or not, and to measure their absolute velocity – even if it, in the Lorentz ether, exists.

- So now everything is prepared to propose the relativistic argumentation in favor of the spacetime interpretation: That, once we cannot identify absolute rest, we cannot assume that it exists. So, we should, instead, accept the existence of a whole four-dimensional spacetime, instead of the three-dimensional space we see. Of course, proponents of this interpretation will find much better arguments in favor of accepting it, so that I will leave this here. My point is that the main physical argument against the preferred frame – the impossibility to identify it by observation – has been already established, and has been established without using anything in contradiction with common sense.

For a first version of an introduction to relativity based on this concept see [15]

X. CONCLUSIONS

We have presented here a generalization of the Lorentz ether interpretation of special relativity to the Einstein equations of general relativity in harmonic coordinates. The harmonic coordinate conditions are interpreted as continuity and Euler equations of the ether, with the gravitational field defining density, velocity and stress tensor of the ether.

We have discussed various aspects of the ether interpretation, as well as of the purpose of different interpretations of a physical theory. We conclude that to know different interpretations is useful for very different groups. In particular, for researchers interested in the developing more fundamental theories, different interpretations, with different problems to be solved, provide different starting points for their theory development. For working physicists, to know different interpretations allows them to distinguish the physical content, shared by all interpretations, from metaphysical aspects which can differ. Moreover, different interpretations suggest them different intuitions. Which intuitions appear to be helpful, and which appear, instead, misleading is worth to be evaluated.

We have found several aspects where the intuitions connected with the ether interpretation seem more sound and less misleading. So, it seems reasonable to expect that what Bell has observed for teaching special relativity remains valid in regard of a classical ether interpretation of gravity too:

For students, learning different interpretations may appear helpful for understanding the theory. The different interpretations give different ways to learn the theory. Here, the ether interpretation may be helpful because it essentially reduces the necessity to change metaphysical ideas. Learning the new mathematics of special and general relativity can be separated from learning spacetime metaphysics.

Moreover, with the ether interpretation there is no need to learn or to accept spacetime metaphysics at all. This may become important for all those who have problems with understanding spacetime metaphysics, or with accepting the necessity to use them, and can prevent them from becoming "ether cranks".

Last but not least, the Lorentz ether interpretation of the Einstein equations allows, in a much better way, to identify the points where the ether concept differs from the spacetime concept, with the background independence as the key difference.

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