VIOLATIONS OF THE RTG CAUSALITY CONDITION IN HOMOGENEOUS UNIVERSE SOLUTIONS OF RTG

I. SCHMELZER

Abstract. We consider some examples of homogeneous universe solutions of the equations of the relativistic theory of gravity which violate the causality condition of this theory.

1. Introduction

The aim of this paper is to consider solutions of the equations of the "relativistic theory of gravity" (RTG) [2], [3] which violate the causality condition of this theory. It is well-known (see [3]) that the causality condition is not fulfilled automatically and should be considered as an additional condition for the selection of physically meaningful solutions. The consideration of solutions which have to be rejected according to the RTG causality conditions is, nonetheless, very interesting for the following two reasons: First, the "general Lorentz ether theory" (GLET) [5], a metric theory of gravity with preferred frame, leads, for an appropriate choice of the signs of the constants of the theory, to the same set of equations as RTG. But GLET has a different, weaker causality condition – the preferred time has to be timelike. To distinguish the two theories, it is, therefore, interesting to consider solutions of the common equations which violate the stronger RTG causality condition. Second, it seems interesting to discuss the RTG causality condition from point of view of the initial value problem: What happens if the initial values $g_{\mu\nu}(x, t_0)$ fulfill the causality condition, but the future evolution according to the RTG evolution equations leads to violations of the causality condition? The consideration of the homogeneous universe is especially interesting from this point of view: The solutions $a(\tau)$ are oscillating, and the causality condition reduces to a simple inequality $a(\tau) \leq \beta$. Thus, even if the causality condition is fulfilled for a_{min}, it may be violated for a_{max}.

We will see in this paper that this, indeed, happens. Moreover, it happens for physically interesting initial values $a(\tau_0)$, $\rho(\tau_0)$ and reasonable values of the material law $p = p(\rho)$.

2. Homogeneous universe equations of RTG

The homogeneous isotropic universe ansatz in RTG (following [2]) is given by the harmonic metric

$$ds^2 = c^2 a^6(t) dt^2 - \beta^4 a^2(t)(dx^2 + dy^2 + dz^2) \tag{1}$$

in coordinates where the background metric is defined by

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - (dx^2 + dy^2 + dz^2). \tag{2}$$

The RTG causality condition requires that the light cone of the effective metric ds^2 is inside the light cone of the background metric $d\sigma^2$. For our ansatz, the condition depends on the scaling factor β:

$$a(t) \leq \beta. \tag{3}$$

Proper time τ is defined by $d\tau = a^3 dt$, so that we obtain, in proper time, the metric

$$ds^2 = c^2 d\tau^2 - \beta^4 a^2(\tau)(dx^2 + dy^2 + dz^2) \tag{4}$$

We obtain the following set of equations ([3], p.129):

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) - \frac{1}{6}(mc)^2\left(1 - \frac{1}{a^6}\right) \tag{5}$$

$$\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 = \frac{8\pi G}{3}\rho(\tau) - \frac{1}{12}(mc)^2\left(2 - \frac{3}{a^2\beta^4} + \frac{1}{a^6}\right). \tag{6}$$

The conservation law $\nabla T^{\mu\nu} = 0$ for the matter gives

$$-3\left(\rho + \frac{p}{c^2}\right)\frac{1}{a}\frac{da}{d\tau} = \frac{d\rho}{d\tau}. \tag{7}$$

with some equation of state $p = p(\rho)$.

For our considerations, it is interesting to consider the difference between (5) and (6):

$$\frac{d^2 \ln a}{d\tau^2} = -4\pi G\left(\rho + \frac{p}{c^2}\right) - \frac{(mc)^2}{4a^6\beta^4}\left(a^4 - \beta^4\right). \tag{8}$$

This equation shows an interesting connection with the vacuum solution $a(\tau) = \beta = 1$. For $m = 0$ this reduces to the Einstein equations for a flat FRW universe. Instead, a non-zero "graviton mass" $m \neq 0$ establishes a connection with the background metric $d\sigma^2$. But the connection does not give a hard boundary $a(\tau) \leq \beta$, but, instead, only an acceleration of $a(\tau)$ toward the vacuum solution $a(\tau) = \beta$.

2.1. **Oscillating solutions without matter.** At first, let's consider the case $\rho = 0$. Equation (6) requires

$$2 - \frac{3}{a^2\beta^4} + \frac{1}{a^6} \leq 0 \tag{9}$$

This expression reaches its minimum value at $a = \beta$. For $\beta = 1$ this gives the constant RTG vacuum solution $a(\tau) = \beta = 1$. For $\beta > 1$ we have no solutions, but for $\beta < 1$ we obtain a region $[a_{min}, a_{max}]$ where (9) is fulfilled, with $a_{min} < \beta < a_{max}$. The boundaries a_{min}, a_{max} define values where $da/d\tau = 0$ according to (6), and with $d^2a/d\tau^2 > 0$ resp. < 0 according to (8). As a consequence, we obtain solutions oscillating between a_{min} and a_{max}, which violate the RTG causality condition near a_{max}, but not near a_{min}.

This allows to define an initial value problem $a(\tau_0) = a_{min}, \rho(\tau_0) = 0$ so that the solution fulfills the causality conditions in some environment of τ_0 but violates it later. Thus, evolution in time does not preserve the causality condition in RTG.

Moreover, solutions of this type appear, in the limit $\beta \to 1$ from below, in an arbitrary small environment of the vacuum solution $a(\tau) = \beta = 1$.

2.2. **Causality-violating solutions for arbitrary density and graviton mass.** Let's consider now the case of small $\beta < 1$, but with nonzero mass $m \neq 0$. For this purpose, we fix a value of β and some initial values $a(\tau_0) = a_0$, $\rho(\tau_0) = \rho_0$. Then, the material law $p = p(\rho)$, together with equation (7), uniquely defines the density ρ as a function $\rho = \rho(a)$, with $\rho(a_0) = \rho_0$. The resulting functions $\rho(a)$, $p(\rho(a))$ we use to rewrite equation (8) as a second order equation of only one independent function $a(\tau)$:

$$\frac{d^2 \ln a}{d\tau^2} = -F(a) - \frac{(mc)^2}{4a^6\beta^4}\left(a^4 - \beta^4\right), \tag{10}$$

with $F(a) = 4\pi G\left(\rho(a) + \frac{p(\rho(a))}{c^2}\right)$. We assume that $F(a) > 0$ and decreases with increasing a. To obtain the RTG solution for given values of m and β, we have to use equation (6) at τ_0 to fix the initial value $v_0 = \dot{a}(\tau_0)$ and to use these initial values a_0, v_0 in equation (10).

In the GR limit $m = 0$, we obtain from (6) the initial value v_{GR} and a solution $a_{GR}(\tau)$ with this initial value which increases without boundary: $a_{GR}(\tau) \to \infty$ for $\tau \to \infty$.

Now, let's consider the case where

$$2 - \frac{3}{a_0^2\beta^4} + \frac{1}{a_0^6} \leq 0 \tag{11}$$

This requires $\beta < 1$. In this case, for a given $a_0 < 1$ (for $a_0 \geq 1$ the RTG causality condition is violated already for the initial values) it can always be reached by taking β sufficiently close to a_0. In this case, the solution always violates the RTG causality condition. Indeed, equation (6) gives in this case an initial value $v_0 > v_{GR}$. This allows to prove that $a(\tau)$ reaches the value β with $\dot{a}(\tau) > 0$, and, therefore, violates the RTG causality condition.

Indeed, assume $a(\tau) < \beta$. Because $a(\tau_0) = a_{GR}(\tau_0)$, $\dot{a}(\tau_0) > \dot{a}_{GR}(\tau_0)$, there will be some $\tau_1 > \tau_0$ so that $a(\tau_1) > a_{GR}(\tau_1)$. Because a_{GR} reaches β, $a(\tau)$ has to intersect $a_{GR}(\tau)$. We consider the first intersection $\tau_2 > \tau_1$ with $a(\tau_2) = a_{GR}(\tau_2)$. Here, $\dot{a}(\tau_2) \leq \dot{a}_{GR}(\tau_2)$. Thus, $\dot{\ln a}(\tau) - \dot{\ln a}_{GR}(\tau)$ decreases between τ_0 and τ_2. Thus, there should be some intermediate value τ_c with $\ddot{\ln a}(\tau_c) - \ddot{\ln a}_{GR}(\tau_c) < 0$. At this point we have $a(\tau_c) > a_{GR}(\tau_c)$, therefore $F(a(\tau_c)) < F(a_{GR}(\tau_c))$, and because of $a(\tau_c) < \beta$ the mass term of (10) is positive. Therefore, the comparison of (10) for $a(\tau_c)$ and $a_{GR}(\tau_c)$ gives $\ddot{\ln a}(\tau_c) > \ddot{\ln a}_{GR}(\tau_c)$, thus, a contradiction. Thus, $a(\tau)$ has to reach β. Equation (6) at $a = \beta$ gives a nonzero value for $\dot{a}$. Indeed, $\rho > 0$, and the mass term reaches its minimum at $a = \beta$ and was already negative at a_0.

Thus, for every initial value $a_0 < 1$, ρ_0 and every mass m we can find a value of β so that the solution violates the RTG causality condition in some future. This value of β can be taken independent of m and ρ_0 – all we need is (11).

2.3. **Causality-violating solutions for large scaling factors.** What about large values of β? First, for fixed values of a_0, ρ_0 and m we can, by increasing β, always find solutions which preserve RTG causality. On the other hand, there is no large enough value of β which would give us a warranty. Instead, for any fixed a_0, ρ_0, and β, for small enough m RTG causality will be violated.

This would follow simply from the continuity of the limit $m \to 0$ of the RTG equations. But, because the continuity of the limit $m \to 0$ of the RTG equations has been questioned in the literature [1], we prefer not to rely on it here and to prove this using a modification of the proof in the last section. The problem is that without (11), the initial value of $\dot{a}(\tau_0)$ will be lower than v_{GR}, thus, comparison with $a_{GR}(\tau)$ does not help us. But can construct an appropriate replacement for a_{GR}: A solution $a_r(\tau)$ of the same equation (10), also for $m = 0$ and with $a_r(\tau_0) = a_0$, but a smaller initial value of the derivative $\dot{a}_r(\tau_0) = v_r < v_{GR}$. The value of v_r should be taken such that $a_r(\tau)$ can be, again, used as a lower bound for $a(\tau)$. For this purpose, it is sufficient that it also reaches β. To construct such a v_r it is sufficient to use the continuous dependence of the solution of ordinary differential equations from their initial values. For some value τ_e with $a_{GR}(\tau_e) > \beta$ this gives an environment $|v_r - v_{GR}| < \delta$ of initial values so that $a_r(\tau_e) > \beta$ too. We take one $v_r < v_{GR}$ from this environment.

Now, we can take a_r as a lower bound for initial values v with $v_{GR} > v > v_r$ of equation (10) with nonzero mass. For a given v, we can use (6) to compute a corresponding mass m. As a consequence, for sufficiently small m we also obtain a violation of the RTG causality condition.

2.4. **Observability of a suffiently small scaling factor.** A sufficiently small value of the scaling factor β would lead to observable effects. The additional term is similar to a homogeneous dark matter term with material equation

$$p(\rho) = -\frac{1}{3}\rho. \tag{12}$$

Especially, if this term dominates the other right hand side terms in (6), we obtain

$$a'(\tau) \approx const. \tag{13}$$

Such a term gives, therefore, observable effects. Moreover, it may be useful in our universe. Indeed, a non-accelerated universe with $a'(\tau) \approx const$ is much closer to the SN1a data than the decelerating universe predicted by GR with standard matter only. Thus, if the errors of the SN1a observations appear to be larger than assumed, so that a universe with $a'(\tau) \approx const$ becomes viable, RTG combined with an appropriate value of β would be viable even without a quintessence term.

On the other hand, if we require the RTG causality condition, the very small value of m in our universe leads to a very large minimal value of β, much too large to lead to observable effects.

3. Discussion

The solutions we have found in this paper are nice, bounded, smooth solutions of the equations of motions common for RTG and GLET. They fulfill the weaker causality condition of GLET, which is connected with absolute time in this theory, and, therefore, do not contain any closed causal loops and other causal artefacts.

Nonetheless, they violate the causality conditions of RTG, and are, therefore, inappropriate as solutions of RTG. Moreover, the violation of the RTG causality condition cannot be detected from the initial values: All the solutions with causality violations considered here have moments τ where the causality condition is fulfilled. This makes the rejection of such solutions problematic. To decide about acceptability of a given set of initial conditions, we have to know the whole solution at future times.

One solution of such a problem is to consider violations of the causality condition as an event where the boundary of applicability of the theory has been reached, and a better, more fundamental, physical theory is necessary to predict what happens. This way to solve such a problem is very natural in the case of GLET, which has a weaker causality condition: The preferred time coordinate T should be timelike. In the condensed matter (ether) interpretation of this theory this corresponds to the condition $\rho_{ether}(X,T) > 0$. Here, it is clear that, at least in principle, it is possible that in some future T_1 a state with $\rho_{ether}(X,T_1) = 0$ may be reached, even for initial conditions $\rho_{ether}(X,T_0) > 0$ everywhere. As well, it is clear, that at this point the condensed matter approximation is no longer applicable, despite the fact that some continuation of the solution into the domain with $\rho_{ether}(X,T) < 0$ exists.

But in the case of RTG such an interpretation seems not natural. Indeed, violations of the RTG causality condition happen in arbitrary close environments of the simplest possible solution at all, the vacuum solution $a(\tau) = \beta = 1$. For a reasonable physical theory, a small variation in the initial conditions for the vacuum solution of type $a(\tau_0) = 1 - \delta$, $\beta = 1 - \varepsilon$ should be unproblematic, but, as we have seen, it leads in RTG to solutions violating the causality condition. Instead, in GLET the distance between the vacuum solution with $\rho_{ether}(X,T) = 1$ and the causality violating situation with $\rho_{ether}(X,T) = 0$ is sufficiently large.

Based on these considerations we suggest that the RTG causality condition should be abandoned as unphysical.

References

[1] A. Godizov, Numerical spherically symmetric static solution of the RTG equations outside the matter, gr-qc/0612122

[2] A.A. Logunov et.al., Theor.Math.Phys. 69 Nr 3, 1986, A.A. Logunov, Theor.Math.Phys. 70 Nr 1, 1987

[3] A.A. Logunov, Relativistic theory of gravity (in russ.), Nauka, 2006

[4] A.A. Logunov, M.A.Mestvirishvili, Yu.V.Chugreev, Graviton mass and evolution of a Friedman universe, Theor.Math.Phys. 74, 1-20 (1988)

[5] I. Schmelzer, General ether theory – a metric theory of gravity with condensed matter interpretation, Proc. of the XXII international workshop on high energy physics and field theory, Protvino, June 23-25, 1999